Shell-model calculations in ^{132}Sn and ^{208}Pb regions with low-momentum interactions

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Abstract.

We discuss shell-model calculations based on the use of low-momentum interactions derived from the free-space nucleon-nucleon potential. A main feature of this approach is the construction of a smooth potential, $V_{\rm low-k}$, defined within a given momentum cutoff. As a practical application of the theoretical framework, we present some selected results of our current study of nuclei around doubly magic ^{132}Sn and ^{208}Pb which have been obtained starting from the CD-Bonn potential. Focusing attention on the similarity between the spectroscopy of these two regions, we show that it emerges quite naturally from our effective interactions without use of any adjustable parameter.

1. Introduction

In the last decade, shell-model calculations employing realistic effective interactions derived from modern nucleon-nucleon (NN) potentials have entered the main stream of nuclear structure theory [1]. As is well known, the first problem one is confronted with in this kind of calculations is the strong short-range repulsion contained in the bare NN potential V_{NN}, which prevents its direct use in the derivation of the shell-model effective interaction $V_{\rm eff}$. The most popular way to overcome this difficulty has long been the Brueckner G-matrix method. However, a few years ago a new approach [2] was proposed which consists in deriving from V_{NN} a low-momentum potential, $V_{\rm low-k}$, that preserves the deuteron binding energy and scattering phase shifts of V_{NN} up to a certain cutoff momentum Λ. This is a smooth potential which can be used directly to derive $V_{\rm eff}$, and it has been shown [2, 3] that it provides an advantageous alternative to the use of the G matrix. In this connection, it should be mentioned that $V_{\rm low-k}$ potentials are currently being used in various nuclear theory contexts, such as the study of few-body systems and no-core shell-model calculations [4, 5].

Making use of the $V_{\rm low-k}$ approach, we have recently studied [6, 7, 8, 9] several nuclei beyond doubly magic ^{132}Sn, showing that their properties are well accounted for by a unique shell-model Hamiltonian with single-particle energies taken from experiment and two-body effective interaction derived from the CD-Bonn NN potential [10].

Motivated by the very good results obtained in the ^{132}Sn region and by the existence of a close resemblance [11, 12, 13, 14] between the spectroscopy of this region and that of nuclei around stable ^{208}Pb, we have found it challenging to perform a comparative study of these two

regions [15]. In this paper we present some results from this study, focusing attention on the proton-proton, neutron-neutron and proton-neutron multiplets in the three far-from-stability nuclei ^{134}Te, ^{134}Sn, ^{134}Sb and in their counterparts in the ^{208}Pb region, ^{210}Po, ^{210}Pb, and ^{210}Bi.

We start by giving an outline of the theoretical framework in which our shell-model calculations are performed and then present and discuss our results. A short summary is given in the last section.

2. Theoretical framework

In the framework of the shell model an auxiliary one-body potential U is introduced in order to break up the nuclear Hamiltonian, written as the sum of the kinetic term and the NN potential, into a one-body component H_0, which describes the independent motion of the nucleons, and a residual interaction H_1. Namely,

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} = T + V_{NN} = (T + U) + (V_{NN} - U) = H_0 + H_1. \tag{1}$$

A reduced model space is then defined in terms of the eigenvectors of H_0 and the diagonalization of the original Hamiltonian H in an infinite Hilbert space is reduced to the solution of an eigenvalue problem for an effective Hamiltonian $H_{\rm eff}$ in a finite space.

The Hamiltonian $H_{\rm eff}$ can be derived by way of the $\hat{Q}$-box folded-diagram expansion (see Ref. [1]). This implies as first step the calculation of the so-called $\hat{Q}$-box, which is made up of an infinite collection of irreducible and valence-linked Goldstone diagrams. Once the $\hat{Q}$-box has been calculated at a given order, the infinite series of the folded diagrams has to be summed up. From this procedure an effective Hamiltonian is obtained containing both one- and two-body components. Usually, only the two-body term $V_{\rm eff}$ is retained, while the one-body contributions, representing the theoretical single-particle energies, are subtracted and replaced with single-particle energies taken from experiment [16].

However, as mentioned in the Introduction no modern NN potential can be used in a perturbative nuclear structure calculation, unless its strong repulsive core is firstly "smoothed out". Here, we do not embark on any discussion of how the effective Hamiltonian is derived, and refer to [1], where a detailed description of the whole procedure can be found. Rather, in the following we focus on the $V_{\rm low-k}$ approach. We first outline the essential steps for the derivation of the $V_{\rm low-k}$ based on the Lee-Suzuki similarity transformation method [17], and then describe its main features.

Let us consider the similarity transformation on the Hamiltonian (1)

$$\mathcal{H} = X^{-1} H X, \tag{2}$$

where the operator X is defined in the whole Hilbert space. We now introduce a cutoff momentum Λ that separates fast and slow modes to the end of deriving from the original V_{NN} a low-momentum potential satisfying a decoupling condition between the low- and high-momentum spaces.

The low-momentum space is specified by

$$P = \int d\boldsymbol{p} \mid \boldsymbol{p}\rangle\langle \boldsymbol{p} \mid \ , \quad p \leq \Lambda \tag{3}$$

where $\boldsymbol{p}$ is the two-nucleon relative momentum, and the decoupling equation reads

$$Q\mathcal{H}P = 0, \tag{4}$$

with $Q = 1 - P$ being the complementary fast-mode space. The low-momentum Hamiltonian is then given by

$$H_{\rm low-k} = P\mathcal{H}P, \tag{5}$$

and it can be easily proved that its eigenvalues are a subset of the eigenvalues of the original Hamiltonian.

There are, of course, different choices for the transformation operator X. We take

$$X = e^{\omega}, \tag{6}$$

where the wave operator ω satisfies the conditions:

$$\omega = Q\omega P, \tag{7}$$

$$P\omega P = Q\omega Q = P\omega Q = 0, \tag{8}$$

the former implying that

$$X = 1 + \omega. \tag{9}$$

From Eq. (5) the low-momentum potential $V_{\rm low-k}$ can be defined as

$$V_{\rm low-k} = H_{\rm low-k} - PTP. \tag{10}$$

Employing transformation (9), this equation is written as

$$V_{\rm low-k} = PV_{NN}P + PV_{NN}Q\omega, \tag{11}$$

while Eq. (4) becomes

$$QV_{NN}P + QHQ\omega - \omega PHP - \omega PV_{NN}Q\omega = 0. \tag{12}$$

The solution of Eq. (12) gives the value of ω needed to obtain $V_{\rm low-k}$.

This decoupling equation can be solved by means of the iterative technique for non-degenerate model spaces proposed in [18], which is now sketched. We define the operators:

$$\begin{aligned} p(\omega) &= PHP + PHQ\omega, & (13)\\ q(\omega) &= QHQ - \omega PHQ, & (14) \end{aligned}$$

in terms of which one can write

$$\begin{aligned} x_0 &= -(QHQ)^{-1}QHP \ , \\ x_1 &= q(x_0)^{-1}x_0 p(x_0) \ , \\ &\dots \\ x_n &= q(x_0+x_1+...+x_{n-1})^{-1}x_{n-1}p(x_0+x_1+...+x_{n-1}) \ . \end{aligned} \tag{15}$$

Once the iterative procedure has converged, $x_n \to 0$, the operator ω is given by

$$\omega_n = \sum_{i=0}^{n} x_i, \tag{16}$$

In applying this method, we have employed a momentum-space discretization procedure making use of an adequate number of Gaussian mesh points [19].

It is worth mentioning that the equation for $V_{\rm low-k}$ obtained using the similarity transformation of Lee and Suzuki is the same as that one can derive from the T-matrix equivalence approach [1, 2]. This means that the obtained low-momentum potential not only preserves the deuteron binding energy given by the original NN potential, but also its low-momentum ($\leq \Lambda$) half-on-shell T matrix.

The above $V_{\rm low-k}$ is however not Hermitian, which is not convenient for various applications, as for instance its use in the derivation of shell-model effective interactions. This $V_{\rm low-k}$ may be transformed by means of the familiar Schmidt orthogonalization procedure, which leads to a Hermitian new $V_{\rm low-k}$. As suggested in [18], another transformation, based on the Cholesky decomposition of a symmetric and positive definite matrix, can be used to this end. In fact, the matrix $P(1+\omega^{+}\omega)P$, being symmetric and positive definite, admits this decomposition,

$$P(1+\omega^{+}\omega)P = PLL^{T}P, \tag{17}$$

where L is a lower triangular matrix and L^{T} its transpose. Since L is real matrix defined within the P-space, we may write our transformation as

$$Z = L^{T}, \tag{18}$$

and the corresponding Hermitian $V_{\rm low-k}$ is

$$V^{\rm chol}_{\rm low-k} = PL^{T}P(H_0 + V_{\rm low-k})P(L^{-1})^{T}P - PH_0P. \tag{19}$$

In Refs. [20, 1], it has been shown that this Hermitian interaction, as well all the family of Hermitian interactions which can derived from $V_{\rm low-k}$ by means of different transformations, preserve the full-on-shell T matrix, and consequently the phase shifts of the original V_{NN}.

The so-obtained $V_{\rm low-k}$ is a smooth potential that can be used directly within the $\hat{Q}$-box folded diagram theory to derive the shell-model effective interaction. Actually, it represents an advantageous alternative to the G-matrix approach owing to the fact that it does not depend either on the energy or on the model space. This is at variance with the G matrix, which is defined in the nuclear medium. In this connection, it is worth mentioning that the merit of the $V_{\rm low-k}$ within the context of realistic shell-model calculations has been assessed by several studies evidencing that $V_{\rm low-k}$ results are as good, or even slightly better than, the G-matrix ones [2, 3].

Finally, it is a remarkable feature of the $V_{\rm low-k}$ approach that different NN potentials lead to low-momentum potentials, which are quite similar to each other [1, 21].

3. Two-valence particle nuclei in ^{132}Sn and ^{208}Pb regions

3.1. Outline of calculations

In this paper, we present some results of our current shell-model study of nuclei with two valence nucleons in the ^{132}Sn and ^{208}Pb regions, which have been obtained starting from the CD-Bonn potential renormalized by use of a $V_{\rm low-k}$ with a cutoff momentum of $\Lambda = 2.2$ fm^{-1}.

In our calculations for ^{134}Te, ^{134}Sn, and ^{134}Sb we assume that the the valence protons occupy the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ of the $50-82$ shell, while for the neutrons the model space includes the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the $82-126$ shell. Similarly, for ^{210}Po, ^{210}Pb, and ^{210}Bi we take as model space for the valence protons the six levels of the $82-126$ shell and let the valence neutrons occupy the seven levels $1g_{9/2}$, $0i_{11/2}$, $0j_{15/2}$, $2d_{5/2}$, $3s_{1/2}$, $1g_{7/2}$ and $2d_{3/2}$ of the $126-184$ shell. As regards the adopted single-particle neutron and proton energies, they can be found in Refs. [6] and [22] for ^{132}Sn and ^{208}Pb, respectively.

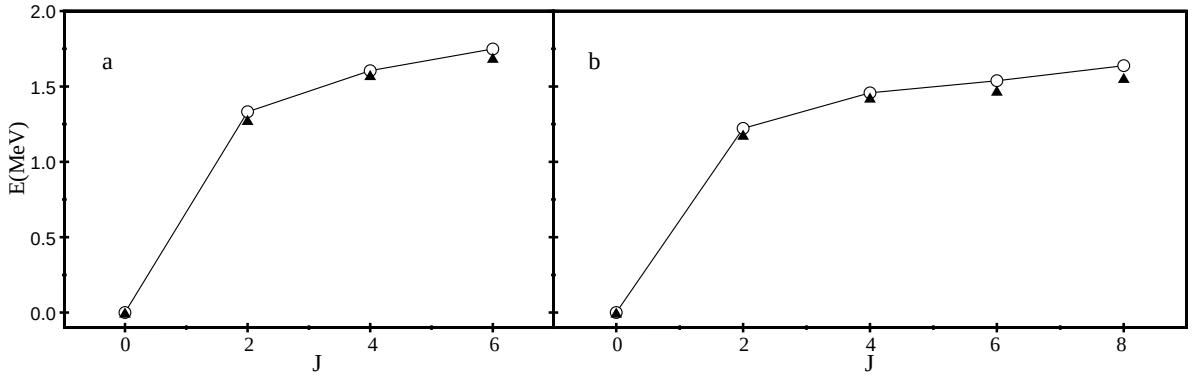

Figure 1. (a) Proton-proton $(\nu f_{7/2})^2$ multiplet in ^{134}Te.(b) Proton-proton $(\nu g_{9/2})^2$ multiplet in ^{210}Po. The theoretical results are represented by open circles while the experimental data by solid triangles.

As mentioned in section 2, the two-body matrix elements of the effective interaction are derived within the framework of the $\hat{Q}$-box folded-diagram expansion. We include in the $\hat{Q}$-box all diagrams up to second order in the interaction, given by the $V_{\mathrm{low-k}}$ potential plus the Coulomb force for protons. These diagrams are computed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. The oscillator parameter is 7.88 MeV for $A = 132$ region and 6.88 MeV for the $A = 208$ region, as obtained from the expression $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. The calculations have been performed by using the NUSHELLX code [23].

3.2. Results

In figures 1, 2, and 3, we present the experimental [24, 25] and calculated excitation energies of the lowest states in the nuclei with two-proton, two-neutron, and one proton-one neutron beyond doubly magic ^{132}Sn and ^{208}Pb. We see that the agreement between theory and experiment is very good for all six nuclei considered, the discrepancies being well below 100 keV for most of the states.

All the calculated states reported in these figures are dominated by a single configuration, whose percentage ranges from 80% to 100%. In particular, they correspond to the proton-proton multiplets $(\pi g_{7/2})^2$ and $(\pi h_{9/2})^2$ in ^{134}Te and ^{210}Po, to the neutron-neutron multiplets $(\nu f_{7/2})^2$ and $(\nu g_{9/2})^2$ in ^{134}Sn and ^{210}Pb, and to the proton-neutron multiplets $\pi g_{7/2}\nu f_{7/2}$ and $\pi h_{9/2}\nu g_{9/2}$ in ^{134}Sb and ^{210}Bi.

These figures evidence the striking resemblance between the behavior of the multiplets in the three pairs of counterpart nuclei, ^{134}Te and ^{210}Po, ^{134}Sn and ^{210}Pb, ^{134}Sb and ^{210}Bi. We may only note that the curves relative to the ^{208}Pb neighbors are generally located slightly below those for the counterpart nuclei in ^{132}Sn region. In particular, from figures 3a and 3b we see that the two proton-neutron multiplets show a sizable energy gap between the 2^- state and the nearly degenerate 0^- and 1^- states, as well as a distinctive staggering, with the same magnitude and phase, between the odd and the even members starting from the 3^- state. As regards the two-identical-particle multiplets, figures 1 and 2 show four curves having all the same shape. It is worth noting, however, that the curves for the two-valence-proton nuclei are located in an energy interval larger than that pertaining to the two-valence-neutron nuclei. This means that in both ^{132}Sn and ^{208}Pb regions a weakening of the pairing gap exists for nuclei with two-valence neutrons with respect to those with two-valence protons.

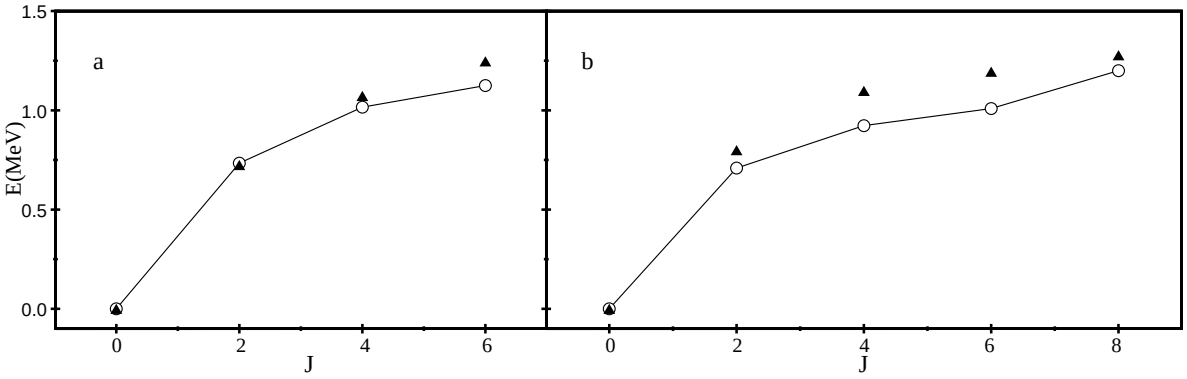

Figure 2. (a) Neutron-neutron $(\pi g_{7/2})^2$ multiplet in ^{134}Sn.(b) Neutron-neutron $(\pi h_{9/2})^2$ multiplet in ^{210}Pb. The theoretical results are represented by open circles while the experimental data by solid triangles.

It is worth mentioning that the resemblance between ^{132}Sn and ^{208}Pb regions was first pointed out by Blomqvist[11], who noticed that every ^{132}Sn single-proton and -neutron level, characterized by quantum numbers (nlj), has its counterpart around ^{208}Pb with quantum numbers $(nl+1j+1)$. However, until recent years the scarcity of information for nuclei around ^{132}Sn, which lies well away from the stability line, has prevented a detailed comparative study of the two regions. Nowadays, new data have become available which support the similarity between their spectroscopies. In our calculation, this similarity emerges quite naturally from our effective interaction which we have derived from a realistic NN potential.

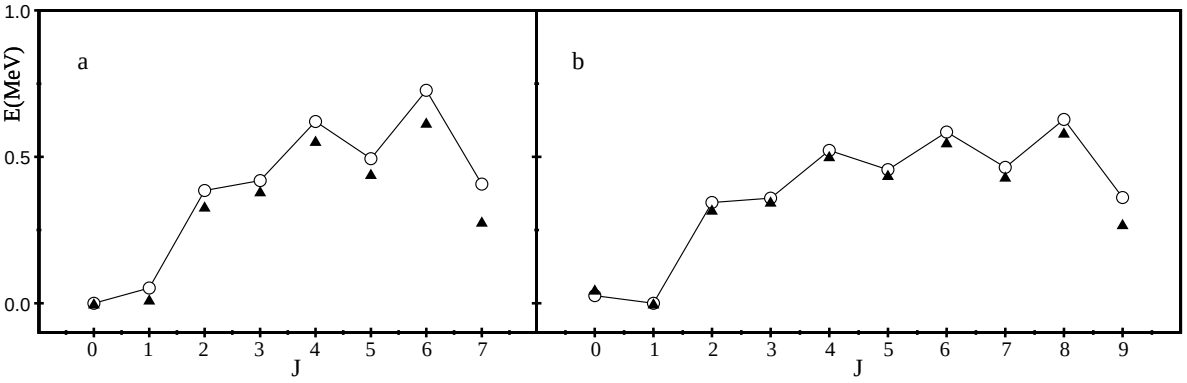

Figure 3. (a) Proton-neutron $\pi g_{7/2}\nu f_{7/2}$ multiplet in ^{134}Sb.(b) Proton-neutron $\pi h_{9/2}\nu g_{9/2}$ multiplet in ^{210}Bi. The theoretical results are represented by open circles while the experimental data by solid triangles.

4. Summary

We have briefly discussed here the theoretical framework for realistic shell-model calculations wherein use is made of low-momentum interactions derived from the free NN potential. We have shown how a smooth low-momentum potential $V_{\rm low-k}$ can be constructed, which preserves the deuteron binding energy and scattering phase shifts of the original V_{NN} up to a given momentum cutoff. We have then presented the results of a shell-model study of nuclei around doubly magic

^{132}Sn and ^{208}Pb, focusing attention on proton-proton, neutron-neutron and proton-neutron multiplets. The results obtained for the three nuclei ^{134}Te, ^{134}Sn and ^{134}Sb have been compared with those for ^{210}Po, ^{210}Pb and ^{210}Bi, which are their counterparts in the region of ^{208}Pb. In both cases, a low-momentum effective interaction derived from the CD-Bonn NN potential has been employed. It should be stressed that no adjustable parameter appears in our calculations.

Our results for all six nuclei are in very good agreement with the experimental data and account for the striking resemblance between the behavior of the multiplets in the ^{132}Sn and ^{208}Pb regions. This stimulates further studies to find out whether this resemblance extends beyond the two-valence-particle nuclei.

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